Exercise 1. **(10 points)** Compute the area of the surface S defined by $z = 2\sqrt{x^2+y^2}$, $x \geq 0$, $y \geq 0$, $0 \leq z \leq 1$.

Exercise 2. Let S be the surface defined by $z = 1 - x^2 - y^2$, $z \geq 0$, and let C be the curve $x^2 + y^2 = 1$, $z = 0$ oriented counterclockwise as viewed from above. Consider the vector field $\mathbf{F} = yz\mathbf{i} + y\mathbf{j} + xy\mathbf{k}$. Compute the circulation $\oint_C \mathbf{F} \cdot d\mathbf{r} = \oint_C \mathbf{F} \cdot T \, ds$ of F around C

(a) **(8 points)** directly by parametrizing the curve C

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(b) **(15 points)** using Stokes' theorem.

Stokes'

Exercise 3. **(22 points)** Let S be the surface defined by $z = x^2 + y^2$, $0 \leq z \leq 1$. Notice that S is open from the top. The surface S is oriented with a normal vector n that points away from the z-axis. Consider the vector field $\mathbf{F} = y\mathbf{i} - x\mathbf{j} + z^2\mathbf{k}$. Compute the flux integral $\iint_S \mathbf{F}.n \, d\sigma$ using the divergence theorem.

$z = r^2$, $r^2 < 1$.

Exercise 4. **(15 points)** Solve the IVP

$$\begin{cases} x^3 \dfrac{dy}{dx} - 2 + x^2 y = 0 \\ y(1) = 1. \end{cases}$$

Exercise 5. **(15 points)** Change the following non exact DE to an exact DE and solve it

$$(8 - 7y + x^8 e^x)dx + xdy = 0.$$

Exercise 6. **(15 points)** Find all the real numbers a such that the solution of the following IVP

$$\begin{cases} \dfrac{dy}{dx} = 2xy^2 \\ y(a) = -1 \end{cases}$$

is defined on $\mathbb{R}$.